Black hole thermodynamics: general aspects and Euclidean formalism

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In the early 1970s, it was shown that Einstein's theory implies that black holes satisfy laws of thermodynamics.

0.- $\boxed{T_H = \frac{\hbar \kappa}{2\pi}}$ is uniform over equilibrium horizons

κ = surface gravity $\quad \kappa = \frac{1}{8\pi M}$ for Schwarzschild

1.- $dE = \underbrace{T_H dS}_{\text{heat}} + \underbrace{\Omega dJ + \Phi dQ + \ldots}_{\text{work}}$ $\quad$ E: black hole mass

$S = \frac{A_H}{4G\hbar}$ $\quad$ A_H: area of event horizon

2.- $\Delta S \geq 0$: area of horizon cannot decrease

The formula $\boxed{S = \frac{A_H}{4G\hbar}}$ is the Bekenstein-Hawking entropy formula

It's our main guide to a quantum theory of gravity.

These laws were proven using differential geometry, not the usual thermodynamics or statistical mechanics tools.

$\Rightarrow$ $\boxed{\text{Spacetime geometry encodes thermodynamic behavior}}$

Black holes are thermal systems:

$T_H = \frac{\hbar \kappa}{2\pi}$; bh's radiate like thermal bodies

This formula doesn't involve gravitational dynamics (no G).
It's a kinematic result of quantum fields in the presence of horizons.

The more fundamental formula is

$$S = \frac{A_H}{4G\hbar}$$ is specific to Einstein-Hilbert dynamics.

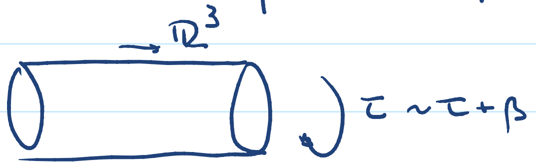
It encodes information about fundamental degrees of freedom of spacetime.

Euclidean formalism for thermal QFT and black holes

QFT at finite temperature is naturally formulated in imaginary time: $t = -i\tau$

Temperature $T = 1/\beta$ means periodicity $\tau \sim \tau + \beta$

$$ds^2 = d\tau^2 + d\vec{x}^2$$



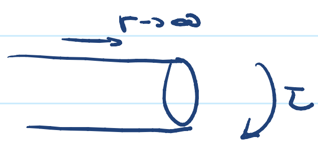
Correlation fns of QFT in this geometry satisfy the KMS condition of thermal equilibrium

$$G(\tau, \vec{x}) = G(\tau + \beta, \vec{x})$$

In a black hole, $t \to -i\tau$

$$ds_E^2 = \left(1 - \frac{2M}{r}\right) d\tau^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega_2$$

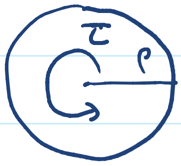
Asymptotically flat $r \to \infty$



Near "horizon" $r \simeq 2M + \frac{\rho^2}{8M}$ $\quad \rho \ll \sqrt{M}$

$$ds_E^2 \simeq \kappa^2 \rho^2 d\tau^2 + d\rho^2 + 4M^2 d\Omega_2 \qquad \kappa = \frac{1}{8M}$$

$\kappa\tau$ is polar angle



Periodic (no conical singu)

$\kappa\tau \sim \kappa\tau + 2\pi$

$$\Rightarrow \tau \sim \tau + \beta \qquad \beta = \frac{2\pi}{\kappa}$$

$$\Rightarrow T_H = \frac{\kappa}{2\pi}$$

A bit more generally, we can use the Euclidean method to easily derive an expression for the temperature of static black holes of the form

$$ds_E^2 = f(r)\, d\tau^2 + \frac{dr^2}{g(r)} + \ldots$$

Regularity at $r = r_H$ where $f(r_H) = 0$ requires $\tau \sim \tau + \beta$ such that

$$2\pi = \lim_{r \to r_H} \frac{C(r)}{R(r)}\bigg|_{r=r_H} = \frac{dC(r)}{dR(r)}\bigg|_{r=r_H}$$

$$= \frac{d\left(\beta\sqrt{f(r)}\right)}{\frac{1}{\sqrt{g(r)}}\, dr}\Bigg|_{r=r_H} = \frac{\beta}{2}\sqrt{\frac{g(r)}{f(r)}}\, f'(r)\bigg|_{r=r_H}$$

$$\Rightarrow T = \beta^{-1} = \frac{1}{4\pi}\sqrt{\frac{g(r)}{f(r)}}\, f'(r)\bigg|_{r=r_H}$$

Correlation functions of quantum fields in a black hole satisfy the KMS thermal periodicity in imaginary time.

Quantum fields in a black hole background are thermally excited. This includes also the graviton field.