Black hole entropy from Euclidean quantum gravity

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BH entropy from Euclidean Quantum Gravity (Gibbons-Hawking)

$$Z[\beta] = \int \mathcal{D}g \, e^{-I_E[g]}$$

$I_E[g]$: classical Euclidean action

sum over geometries that are periodic in imaginary time $\Delta\tau = \beta$

$$I_E(g) = -\frac{1}{16\pi G}\int_{\mathcal{M}} d x^5 \sqrt{g}\, R - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} \sqrt{h}\, K$$

Einstein-Hilbert bulk term

York-Gibbons-Hawking boundary term.
Required to have well-defined variations w/ Dirichlet bc's (fixed metric at $\partial\mathcal{M}$)

h_{ij} : metric induced at $\partial\mathcal{M}$, $h = \det h_{ij}$

K_{ij} : extrinsic curvature of $\partial\mathcal{M} \subset \mathcal{M}$ $\quad K = h^{ij} K_{ij}$

Ill-defined:
- UV problems: non-renormalizability
- IR problems: unboundedness under conformal transformations (there are ways around this)
- what geometries/topologies/signatures... are allowed?

Evaluate it in the saddle-point approximation:

$$Z[\beta] \approx e^{-I_E[g_{cl}]}$$

g_{cl} : classical solution to the field equations of $I_E[g]$ $\quad \frac{\delta I_E}{\delta g}[g_{cl}] = 0$

with g_{cl} periodic in $\tau \sim \tau + \beta$

This is like a WKB approximation to quantum mechanics, in the semiclassical limit.

There may be more than one saddle contributing. The dominant one is the one with smallest $I_E[g_{cl}]$

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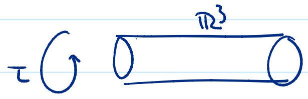
$$F = -\frac{1}{\beta}\log Z \approx \frac{1}{\beta} I_E \qquad \text{(using the saddle-point result)}$$

$$E = -\frac{\partial \log Z}{\partial \beta} \approx \partial_\beta I_E$$

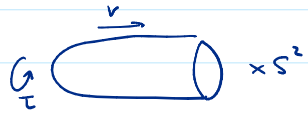
$$S = -(\beta\partial_\beta - 1)\log Z \approx (\beta \partial_\beta - 1)\, I_E$$

For AF spaces w/ $\tau \sim \tau + \beta$, we have at least two different saddle point geometries:

Euclidean flat space $\quad S^1 \times \mathbb{R}^3$



Euclidean Schw black hole $\quad S^2 \times \mathbb{R}^2$



↳ horizon is a fixed point of Euclidean time evolution

The action I_E is infinite, since space is infinite. To deal with this:

- First, regularize w/ cutoff at $r = R$ (large radius)
- Subtract $\quad I_E - I_E^0 \qquad I_E^0$ action of a reference background state (~vacuum)

  Ensure that boundary conditions are the same for $g_{\mu\nu}$ and $g^0_{\mu\nu}$.

  $g_{\mu\nu}$: Euc Schw with $\beta = 8\pi M$

  $g^0_{\mu\nu}$: Eucl Mink

  $\Rightarrow$ finite $\quad I_E - I_E^0 \approx \beta F(\beta) \quad$ for Schw bh

This can indeed be carried out and one finds $F = \frac{\beta}{16\pi}$

so, using the expressions above, $E = \frac{\beta}{8\pi} = M$

$S = 4\pi M^2 = \frac{A_H}{4}$ $\qquad$ which are correct.

But for obtaining only the entropy we can take a shortcut, which avoids the regularization and subtraction.

- For any field in flat space if $\partial/\partial\tau$ is a Killing symmetry

$$I_E = \int d\tau\, d^3x\, \mathcal{L} = \beta \int d^3x\, \mathcal{L} = \beta H \qquad H: \text{hamiltonian}$$

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$$\Big(\mathcal{L} = H + p\underbrace{\dot{q}}_{=0}\Big)$$

$$S = (\beta\partial_\beta - 1) I_E = 0$$

This means that the classical contribution to the entropy in QFT is zero. Entropy in QFT arises at one-loop level

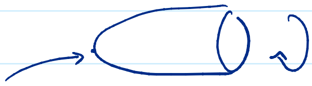
$$\boxed{\begin{array}{l}\text{entropy is a sum over microstates} = \text{Trace over states} \\ \sum_{\text{states}} \bigcirc \to \sim \#\text{ of states of the theory}\end{array}}$$

This result is valid for any Euclidean spacetime where $\partial/\partial\tau$ generates a symmetry in which the action of $\partial/\partial\tau$ is well-defined everywhere, ie without singularities nor fixed-points.
In this case,

$$I_E \propto \beta \Rightarrow S = 0$$

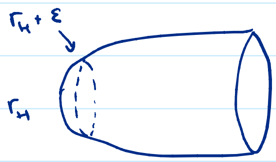
. When there's a black hole horizon (static — can be extended to stationary)

$\dfrac{\partial}{\partial\tau}$ has a fixed point at the Euclidean horizon



At the horizon the Hamiltonian evolution degenerates

Split the integration of the action into two regions: near the horizon (cap), and the rest (cylinder)



The contribution to I_E from $r > r_H + \varepsilon$ is $\propto \beta \Rightarrow$ This region doesn't contribute to S (it would contribute to F and to E, and is the region that requires regularization and subtraction, but S is unaffected by that)

So we only need to compute the Euclidean gravitational action:

$$I_E = -\frac{1}{16\pi G}\int_M d^4x \sqrt{g}\, R - \frac{1}{8\pi G}\int_{\partial M} d^3x \sqrt{h}\, K$$

in the cap region.

For a solution of $R_{\mu\nu} = 0 \Rightarrow R = 0$: The bulk term vanishes.

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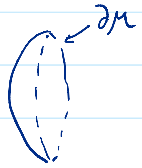
since the volume (and any bulk magnitude) vanishes in the limit $\varepsilon \to 0$.
Only the boundary GHY term will contribute to the entropy.

$$K = \frac{1}{\sqrt{h}}\, n^\mu \partial_\mu \sqrt{h}$$

n^μ : normal to the boundary

$h_{\mu\nu}$: metric induced at bdry

$$I_E^{(\varepsilon)} = -\frac{1}{8\pi G}\, n^\mu \partial_\mu \int_{\partial M} d^3x\, \sqrt{h}$$



$$\simeq -\frac{1}{8\pi G}\, n^\mu \partial_\mu \int_{r \simeq r_H + \varepsilon} d^3x\, \sqrt{h}$$

$$\int d^3x = \int d\tau\, d^2\Omega$$

$$ds^2 \approx \kappa^2 x^2 d\tau^2 + dx^2 + r_H^2\, d\Omega_2 \qquad r - r_H \propto x^2$$

$$\sqrt{h} = \kappa x\, r_H^2\, \omega_2$$

$\omega_2 = \sin\theta$: volume element of S^2 (it could be any transverse space)

Euclidean regularity requires $\kappa = \frac{2\pi}{\beta}$

$$n^x = 1 \qquad n^\mu \partial_\mu = \partial_x$$

$$A_H = 4\pi r_H^2$$

$$I_E^{(\varepsilon)} = -\frac{1}{8\pi G}\, \underset{\uparrow \atop \int d\tau}{\beta}\; \underset{\uparrow \atop \int d^2\Omega}{4\pi}\; \kappa r_H^2\, \partial_x x = -\frac{1}{4G}\, 4\pi r_H^2 = -\frac{1}{4G} A_H$$

$$\Rightarrow \qquad S = (\beta \partial_\beta - 1) I_E = (\beta \partial_\beta - 1) I_E^{(\varepsilon)}$$

$$\Rightarrow \quad \boxed{S = \frac{A_H}{4G}}$$ The correct Bekenstein-Hawking entropy

This derivation clearly localizes the origin of the entropy at the horizon.

Restoring $\hbar$: $\quad S = \frac{A_H}{4G\hbar}$: zero-loop contribution!

$\Rightarrow$ Gravity is some sort of classical collective "mean field", which emerges after statistical averaging over microscopic degrees of freedom, which can capture their number (entropy) w/out revealing their microscopic nature

To some extent, these microscopic degrees of freedom can be seen as "bulk degrees of freedom" in string theory.
But at a more fundamental level, the degrees of freedom are holographic, living in D-1 dimensions (or fewer).